

What is claimed is:

1. A machine learning method, for at least one of data regression and data classification, that is a hybrid of neural net ("NN") analysis and support vector machine ("SVM") analysis, the method comprising:

(1) providing an NN component, having an input layer and a hidden layer and an input vector space, and providing an SVM component, having a feature vector space;

(2) selecting a group of parameters and combinations of parameters and providing a feature space coordinate, in the feature vector space, for each selected parameter and selected parameter combination in the input space;

(3) providing at least one vector of candidate parameter values for each of the group of parameters in the input space;

(4) providing initial values for connection weights between the input layer and the hidden layer for the NN component;

(5) computing hidden layer output signals, corresponding to the connection weight values, for each of the parameter value vectors;

(6) determining an inner product value of a selected number of at least two feature space coordinates;

(7) providing a Lagrange functional using the determined inner product values;

(8) providing at least two constraints, expressed in terms of Lagrange multipliers and input space data;

(9) minimizing the Lagrange functional, subject to at least one selected constraint, to obtain Lagrange multiplier values corresponding to the minimized Lagrange functional;

(10) computing a training error, using the connection weights for the NN component and the Lagrange multiplier values for the SVM component;

(11) when the computed training error is greater than a selected threshold value, changing at least one of the connection weights and repeating steps (5)-(10); and

(12) when the computed training error is not greater than the threshold value, interpreting the NN component with the associated connection weights and the SVM component with the associated Lagrange multipliers as a trained NN/SVM system.

2. The method of claim 1, further comprising:  
providing an optimization method; and  
using the optimization method in at least one of steps (5) through (12) to minimize said training error and to obtain at least one of said connection weight values and said Lagrange multiplier values.

3. The method of claim 1, further comprising determining an optimized design by applying a response surface analysis to said design, using said trained NN/SVM system.

4. The method of claim 3, further comprising providing a selected optimization procedure in determining said optimized design.

5. The method of claim 1, further comprising augmenting said inner product value with at least one user-specified inner product value to said SVM component.

6. The method of claim 1, further comprising:  
providing a collection of N data points in an M-dimensional space for said input space, where  $M \geq 2$  and  $N \geq 2$ , and where each data point is assigned an indicium associated with one of at least first and second mutually exclusive sets;  
and

applying the method of claim 1 for determination of a separation surface in the M-dimensional space that separates the data points into at least first and second mutually exclusive regions that contain substantially all data points in the first set and in the second set, respectively.

7. The method of claim 6, further comprising providing a visually perceptible view of at least a portion of said separation surface in at least two dimensions.

8. A method for design optimization, the method comprising:

(1) providing a group of M parameters that define a design, and providing a vector  $\mathbf{p} = \mathbf{p}_0$  of initial values for each parameter in the group, where M is a selected integer  $\geq 1$ ;

(2) providing data point values for an optimal design at one or more selected location values;

(3) providing an M-simplex in parameter space, centered at a vector location  $\mathbf{p} = \mathbf{p}_0$  and having a selected diameter  $d_0$ ;

(4) providing a design function, and providing a data point value for the first design function at each location value, depending on the choice of the parameter vector  $\mathbf{p}$ , for the parameter vector  $\mathbf{p}_0$  and for each parameter vector corresponding to a vertex of the M-simplex, and for an expanded M-simplex, centered at  $\mathbf{p}_0$ ;

(5) providing a selected first objective function  $\text{OBJ}(\mathbf{p}; \mathbf{p}_0; 1)$ , dependent upon the parameter vector  $\mathbf{p}$  and upon a difference between the optimal design data point value and the first design function data point value at one or more of the location values;

(6) determining a parameter vector  $\mathbf{p} = \mathbf{p}(\min)$  within the expanded M-simplex for which the first objective function attains a minimum value;

(7) computing a selected second objective function  $\text{OBJ}(\mathbf{p}; \mathbf{p}_0; 2)$  for  $\mathbf{p} = \mathbf{p}(\min)$  and for  $\mathbf{p} = \mathbf{p}_0$ ;

(8) when  $\text{OBJ}(\mathbf{p}(\min); \mathbf{p}_0; 2)$  is not less than  $\text{OBJ}(\mathbf{p}_0; \mathbf{p}_0; 2)$ , providing a modified expanded M-simplex, with a modified diameter  $d'$  satisfying  $d_0 < d' < d$  and repeating steps (6) and (7) at least once; and

(9) when  $\text{OBJ}(\mathbf{p}(\min); \mathbf{p}_0; 2)$  is less than  $\text{OBJ}(\mathbf{p}_0; \mathbf{p}_0; 2)$ , determining if  $\text{OBJ}(\mathbf{p}(\min); \mathbf{p}_0; 2)$  is no greater than a selected threshold value;

(10) when  $\text{OBJ}(\mathbf{p}(\min); \mathbf{p}_0; 2)$  is greater than the threshold value, providing a substitute M-simplex, centered at  $\mathbf{p} = \mathbf{p}'_0 = \mathbf{p}(\min)$  with the selected diameter  $d_0$ , and an expanded substitute M-simplex, centered at  $\mathbf{p}'_0$  with the diameter  $d$ , and repeating steps (4), (5), (6), (7), (8) and (9) at least once; and

(11) when  $\text{OBJ}(\mathbf{p}(\min); \mathbf{p}_0; 2)$  is not greater than the threshold value, interpreting the parameter set  $\mathbf{p} = \mathbf{p}(\min)$  as an optimal design set.

9. The method of claim 8, further comprising choosing said first objective function to be

$$\text{OBJ}(\mathbf{p}; \mathbf{p}_0) = \sum_{k=1}^K w_k |f(\mathbf{r}_k; \mathbf{p}) - f(\mathbf{r}_k; \text{opt})|^q,$$

where  $\mathbf{r}_k$  is one of said selected location values,  $f(\mathbf{r}_k; \mathbf{p})$  is one of said design function data point values,  $f(\mathbf{r}_k; \text{opt})$  is one of said optimal design data point values,  $w_k$  is a selected non-negative weight coefficient,  $q$  is a selected positive number, and  $K$  is a selected positive integer.

10. The method of claim 8, further comprising choosing said first objective function to be the same as said second objective function.

11. The method of claim 8, wherein said second objective function depends upon said second design function, upon said parameter vector  $\mathbf{p}$  and upon a difference between said optimal design data point value and a data point value computed using computer simulation of a response of said design, at one or more of said location values.

12. The method of claim 8, further comprising choosing said design function to correspond to pressure on an airfoil at said selected locations on a perimeter of the airfoil.

13. The method of claim 8, further comprising:  
providing data point values for a second optimal design at one or more selected location values; and

applying steps (1)-(11) of claim 8 to obtain an optimal set of design parameters for the second optimal design, where the second optimal design has selected third and fourth objective functions that are independent of said first and second objective functions.